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OPTIMUM DESIGN OF STIFFENED PLATES

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Dedicated to Professor István Páczelt on the occasion of his sixtieth birthday

Abstract. The aim of this paper is to show how to make real structural optimizations on a strong theoretical background. Using stiffened plates one can get a lightweight and stiff structure. Several calculations have been developed for stiffened plates. All of them are approximations: the Massonnet and the Gienke techniques. Cost calculation is also important, due to the expensive welding technologies. Two applications are shown: shipdeck panel and compressed stiffened plate. It is shown that using optimization, one can reduce the total cost of the structure. In countries where fabrication costs are high the number of stiffeners is small and the thickness is large. In countries where fabrication costs are low the number of stiffeners is large and the thickness is small [1, 1999].

Keywords: Structural optimization, stiffened plates, cost calculation

1. Introduction

A stiffened plate has low mass and high bending stiffness. The use of welding made it possible to produce different constructions. To increase the torsional rigidity, cellular plates have been introduced. Stiffened plates can be applied as roof structures of supermarkets, petrol stations, etc. (Figure 1), orthotropic bridge decks (Figure 2), airplane wing structures (Figure 3), ship wall and deck structures (Figure 4), roof structure of tanks (Figure 5) [2, 1966], [3, 1968].

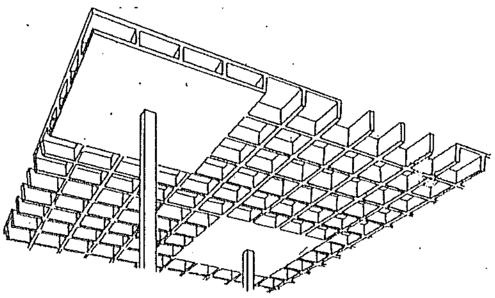

Figure 1.

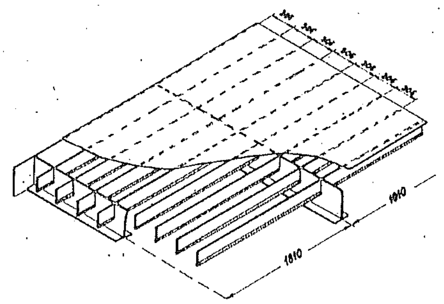

Figure 2.

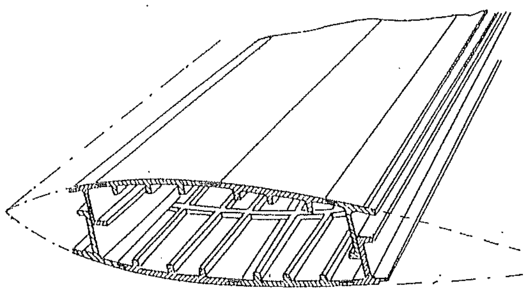

Figure 3.

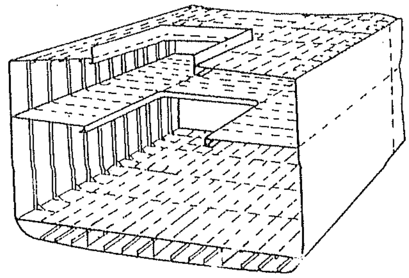

Figure 4.

The following parameters can be varied:
- Different base plate configurations: rectangular, triangular, circular, trapezoidal, etc.,
- Stiffener sections: flat, L-, T-, trapezoidal, etc.,
- Geometry of stiffeners: one-, two directional, one-, two side,
- Technologies: spot-, line welding, riveting, gluing, etc.,
- Loading: static, dynamic, stochastic, uniformly distributed, hydrostatic, concentrated force.

2. Static calculation of stiffened plates

2.1. Grid calculation. If the number of stiffeners is small, the stiffened plate can be divided into beam-like grid structures (Figure 6). This calculation is based on force method. The torsional stiffness can be neglected. The deflections at the nodes should be equal for the two orthogonal beams. The unknown internal forces can be calculated from the deflection equations [4, 1969].

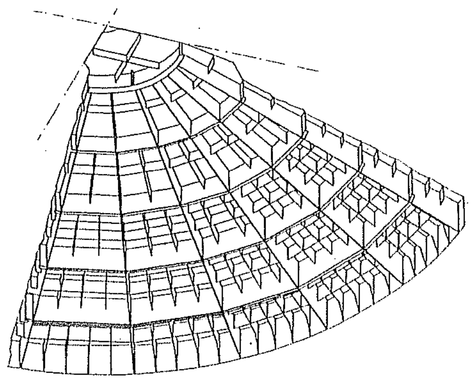

Figure 5.

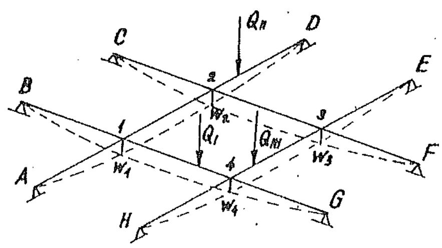

Figure 6.

2.2. Calculation as an anisotropic continuum. The assumptions are as follows:
- elastic stress and deformations,

- deflections are small compared to the thickness of the plate,
- normal stresses orthogonal to the plate can be neglected,
- shear deformations can be neglected,
- stresses from torsion can be calculated from Saint-Venant theory,
- number of stiffeners in both directions is large enough to assume that the effective plate width is equal to the distance between stiffeners.

The stiffness matrix of a plate stiffened on one side in two directions can be formulated from three matrixes: cover plate and stiffeners in x- and y-directions. The reference plane is the mid cover plane. The equilibrium equations concerning the deflections are as follows:

$$(D + D_{bx})\, u'' - z_{Sx} D_{bx} w''' + \frac{1-\nu}{2} D\, \ddot{u} + \frac{1-\nu}{2} D\, \dot{v}' = 0\,, \tag{2.1}$$

$$(D + D_{by})\, \ddot{v} - z_{Sy} D_{by} \dddot{w} + \frac{1-\nu}{2} D\, v'' + \frac{1-\nu}{2} D\, \dot{u}' = 0\,, \tag{2.2}$$

$$\begin{aligned} &z_{Sx} D_{bx} u''' + z_{Sy} D_{by} \dddot{v} - \left(B + B_{bx} + z_{Sx}^2 D_{bx}\right) w'''' - \\ &- \left(B + B_{by} + z_{Sy}^2 D_{by}\right) \ddddot{w} - (2B + B_{xy} + B_{yx})\, \ddot{w}'' + p = 0 \end{aligned} \tag{2.3}$$

where D is the tensional stiffness of the isotropic plate, t_f is the thickness of the cover plate, u, v and w are deflections in x, y and z directions, ν is the Poisson ratio, $z_{Sx} D_{bx}$, $z_{Sy} D_{by}$ are related to the linear moment, $\dot{w}$ and w' are derivatives of w in x and y directions, E is the Young modulus, p is the uniformly distributed load, perpendicular to the cover plate.

Introducing the notations

$$D + D_{bx} = D_x\,, \qquad D + D_{by} = D_y \tag{2.4}$$

the bending stiffnesses of the cover plate and the stiffener in x- and y directions are

$$B_x = B + e_x^2 D + B_{bx} + D_{bx} \left(z_{Sx} - e_x\right)^2 \tag{2.5a}$$

$$B_y = B + e_y^2 D + B_{by} + D_{by} \left(z_{Sy} - e_y\right)^2 \tag{2.5b}$$

where B, B_x, B_y are bending stiffnesses, D_{bx}, D_{by} are tensional stiffnesses of the stiffeners in x- and y directions, e_x, e_y are eccentricities.

Substituting equations (2.4-2.5b) into (2.1-2.3) we obtain

$$\begin{aligned} B_x w'''' + 2\left(B + B_{xy} + B_{yx}\right) \ddot{w}'' + B_y \ddddot{w} + \frac{D}{2} (1-\nu)\, e_y\, \ddot{u}' + \\ + \frac{D}{2} \left[(1+\nu)\, e_x + (1-\nu)\, e_y\right] \dot{v}'' = p \end{aligned} \tag{2.6}$$

For a symmetrically stiffened plate on both sides, $e_x = e_y = 0$ then (2.6) will have a simpler form referred to as Huber equation

$$B_x w'''' + 2\left(B + B_{xy} + B_{yx}\right) \ddot{w}'' + B_y \ddddot{w} = p\,. \tag{2.7}$$

For an isotropic plate $B_x = B_y = B$ and $B_{xy} = B_{yx} = 0$.

2.3. Calculation of eccentrically stiffened plates. For an eccentrically stiffened plate (2.1-2.3), eliminating u and v, gives the following form

$$a_1\frac{\partial^8 w}{\partial x^8}+a_2\frac{\partial^8 w}{\partial x^6\partial x^2}+a_3\frac{\partial^8 w}{\partial x^4\partial y^4}+a_4\frac{\partial^8 w}{\partial x^2\partial y^6}+a_5\frac{\partial^8 w}{\partial y^8}=f(\frac{\partial^4 p}{\partial x^4},\frac{\partial^4 p}{\partial x^2\partial y^2},\frac{\partial^4 p}{\partial y^4}) \tag{2.8}$$

where a_1, a_2, a_3 are parameters and f is the loading function.

The 8th order partial differential equation shows the complexity of the general problem. For a symmetrical structure the equation can be solved by infinite mathematical series. The other solution is to introduce approximations, like reduced stiffnesses, which leads to a Huber equation. This kind of method was developed by [5, 1959] and [6, 1955].

2.3.1. Massonnet technique. The elastic energy of the plate is given by

$$U=\frac{E}{2\left(1-\nu^2\right)}\iiint\limits_{\text{cover plate}}\left(\varepsilon_x^2+\varepsilon_y^2+2\nu\varepsilon_x\varepsilon_y+\frac{1-\nu}{2}\gamma_{xy}^2\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z+$$
$$+\frac{E}{2}\iiint\limits_{\text{stiffeners}}\left(\varepsilon_x^2+\varepsilon_y^2\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z+\frac{1}{2}\iint_A\left(B_{xy}+B_{yx}\right)\dot{w}'\mathrm{d}x\mathrm{d}y \tag{2.9}$$

where $\varepsilon_x, \varepsilon_y$ and γ_{xy} are the strains. The displacements in the x, y and z directions are denoted by u, v and w. Massonnet assumes that

$$u=c_x w', \qquad v=c_y\dot{w}$$

where c_x and c_y are parameters.

Due to the shear stiffness of the cover plate, the eccentricities are less than e_x and e_y.

While we want to determine u and v for a given w, in the equation of U we should consider the parts, which depend on u and v. Solving equation (2.9) we get the reduced Huber equations

$$B_x^* w''''+2H^*\ddot{w}''+B_y^*\ddddot{w}=p\,, \tag{2.10a}$$
$$B_x^*=B_x+(e_x-c_x)^2D_x\,, \tag{2.10b}$$
$$B_y^*=B_y+(e_y-c_y)^2D_y\,, \tag{2.10c}$$
$$2H^*=2B+B_{xy}+B_{yx}+\frac{1-\nu}{2}D(c_x+c_y)^2+2\nu Dc_xc_y\,. \tag{2.10d}$$

This is an iteration procedure: first take an approximation function to w, determination of c_x, c_y calculation the reduced stiffnesses, get a better approximation to w and start a new iteration.

2.3.2. Gienke technique. In order to simplify calculation Gienke suggested considering D_x and D_y as infinitely great quantities. It means that we neglect the deformation of the fibre in gravity center to w:

$$1/D_x = 1/D_y = 0\,, \tag{2.11}$$

$$c_x = e_x\,, \qquad c_y = e_y \tag{2.12}$$

where c_x, c_y are parameters.

The bending stiffnesses are now given by

$$B_x^* * = B_x\,, \qquad B_y^* = B_y \tag{2.13}$$

while for the half torsional stiffness H one can write

$$2H^* = 2B + B_{xy} + B_{yx} + \frac{1-\nu}{2} D(e_x + e_y)^2 + 2\nu D e_x e_y\,. \tag{2.14}$$

The Gienke calculation is less accurate, but simpler than that of Massonnet, because there is no need for iterations.

2.3.3. Navier solution of square stiffened plates subject to bending. We are looking for $w = w(x,y)$ function as a solution of the equation

$$B_x w'''' + 2H^* \ddot{w}'' + B_y \ddot{\ddot{w}} = p(x,y) \tag{2.15}$$

$$2H^* = B + B_{xy} + B_{yx} \tag{2.16}$$

If the plate is a square one and is simply supported, equation (2.15) is associated with the boundary conditions

$$\begin{aligned} w &= 0, \quad m_x = 0 \quad \text{if } x = 0 \text{ and } x = b_x \\ w &= 0, \quad m_y = 0 \quad \text{if } yx = 0 \text{ and } y = b_x \end{aligned}$$

As is well known, the solution assumes the form

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mn} \sin\frac{m\pi x}{b_x} \sin\frac{n\pi y}{b_y} \tag{2.17}$$

where b_x and b_y are the sizes of the plate in x and y directions. We remark that the load can also be given in this form

$$p(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{b_x} \sin\frac{n\pi y}{b_y} \tag{2.18}$$

The coefficients a_{mn} and c_{mn} are related to each other via the equation

$$c_{mn} = \frac{a_{mn}}{\pi^4 (B_x \frac{m^4}{b_x^4} + 2H \frac{m^2 n^2}{b_x^2 b_y^2} + B_y \frac{m^4}{b_y^4})} \tag{2.19}$$

We can get the solution that for uniformly distributed load $p(x,y) = p$

$$a_{mn} = 16p/\pi^2 mn\,. \tag{2.20}$$

3. Cost calculation of stiffened plates

3.1. Fabrication costs. The cost function can be expressed as

$$K = K_m + K_f = k_m \rho V + k_f \sum_i T_i \qquad (i = 1, 2, ..., 7) \tag{3.1}$$

where K_m and K_f are the material and fabrication costs, respectively, k_m and k_f are the corresponding cost factors, ρ is the material density, V is the volume of the structure, T_i are the production times [7, 1997], [8, 1999].

3.2. Welding times. Time for preparation, assembly and tacking is given by

$$T_1 = C_1 \delta \sqrt{\kappa \rho V} \tag{3.2}$$

where δ is a difficulty factor, κ is the number of structural elements to be assembled [9, 1992]. For the welding time one can write

$$T_2 = \sum_i C_{2i} a_{wi}^n L_{wi} \tag{3.3}$$

in which a_{wi} is the weld size, L_{wi} is the weld length and C_{2i} are constants determined by the welding technology. Time for additional fabrication activities such as changing the electrode, deslagging and chipping can be calculated as

$$T_3 = \sum_i C_{3i} a_{wi}^n L_{wi} \tag{3.4}$$

in which [10, 1985] proposed for the constants that $C_3 = (0.2{-}0.4)C_2$ and $C_3 = 0.3C_2$. Neglecting $\sqrt{\Theta_d}$ one obtains

$$T_2 + T_3 = 1.3 \sum_i C_{2i} a_{wi}^2 L_{wi} \tag{3.5}$$

which is a modified formula for $T_2 + T_3$.

Table 1. Applied welding technologies

SMAW	Shielded Metal Arc Welding
SMAW HR	Shielded Metal Arc Welding High Recovery
GMAW-C	Gas Metal Arc Welding with CO2
GMAW-M	Gas Metal Arc Welding with Mixed Gas
FCAW	Flux Cored Arc Welding
FCAW-MC	Metal Cored Arc Welding
SSFCAW (ISW)	Self Shielded Flux Cored Arc Welding
SAW	Submerged Arc Welding
GTAW	Gas Tungsten Arc Welding

Different welding technologies are shown in Table 1.

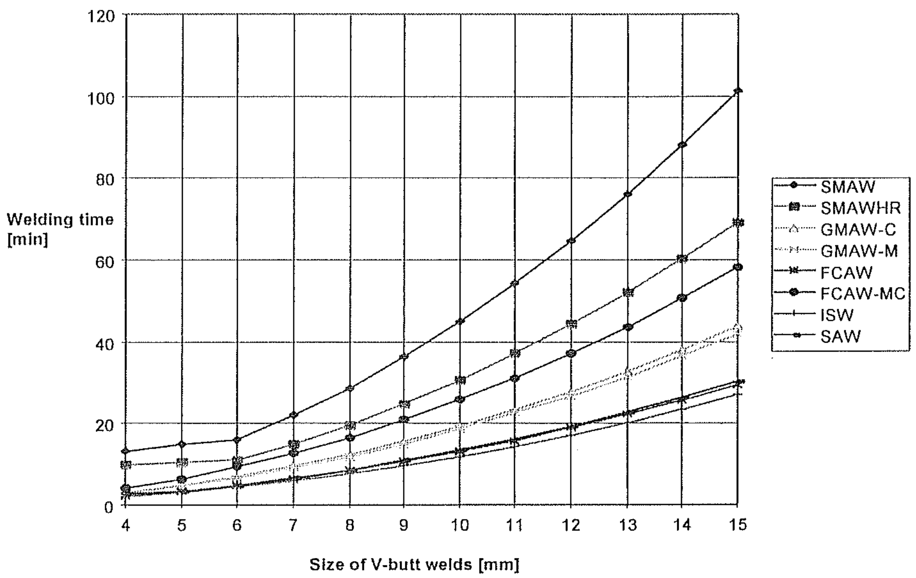

Figure 7. Welding time T_2 (min) for different welding technologies plotted against the weld size a_w (mm) for longitudinal V butt welds downhand position

In Figure 7 data are given for eight welding techniques and for a given weld type.

Using COSTCOMP [11, 1990] software we have calculated the welding time T_2 (min) as a function of weld size a_w (mm) for longitudinal fillet welds, for 1/2 V and V butt welds, for K and X butt welds, for T butt welds, for U and double U butt welds, in downhand position [12, 1990].

The welding time T_2 (min/mm) as a function of weld size a_w (mm) for longitudinal V butt welds is increasing in positional welding, which means not downhand, but vertical or overhead positions. Figure 7 shows that the welding time for longitudinal V butt welds in decreasing order is the highest for SMAW, SMAW-HR, GMAW-C, GMAW-M, FCAW, FCAW-MC, ISW and the lowest for SAW.

3.3. Time for flattening plates. In the catalogues of different companies one can find the times for flattening plates (T_4 [min]) as the function of the plate thickness (t [mm]) and the area of the plate (A_p [mm^2]). The time function can be written in the form:

$$T_4 = \Theta_{de}(a_e + b_e t^3 + \frac{1}{a_e t^4})A_p \,, \tag{3.6}$$

where $a_e = 9.2 * 10^{-4}$ [min/mm^2], $b_e = 4.15 * 10^{-7}$ [min/mm^5], Θ_{de} is the difficulty parameter (Θ_{de}= 1,2 or 3). The difficulty parameter depends on the form of the plate.

3.4. Surface preparation time. Surface preparation means the surface cleaning, painting, ground coat, top coat, sand-spraying, etc. The surface cleaning time can be given in terms of the surface area (A_s [mm^2]) as follows:

$$T_5 = \Theta_{ds} a_{sp} A_s \tag{3.7}$$

where $a_{sp} = 3 * 10^{-6}$ [min/mm^2], Θ_{ds} is a difficulty parameter.

3.5. Painting time. Painting means making the ground and the topcoat. The painting time depends on the surface area (A_s [mm^2]) as follows:

$$T_6 = \Theta_{dp}(a_{gc} + a_{tc})A_s \tag{3.8}$$

where $a_{gc} = 3 * 10^{-6}$ [min/mm^2] , $a_{tc} = 4.15 * 10^{-6}$ [min/mm^2], Θ_{dp} is a difficulty factor, Θ_{dp}=1,2 or 3 for horizontal, vertical or overhead painting.

3.6. Cutting and edge grinding times. Cutting and edge grinding can be done by different technologies, like Acetylene, Stabilized gasmix and Propane with normal and high speed. The cutting time can be calculated also by COSTCOMP. The normal speed acetylene has the highest time and the high speed propane has the smallest cutting time.

The cutting cost function can be formulated as a function of the thickness (t [mm]) and cutting length (L_c [mm]):

$$T_7 = \sum C_{7i} t_i^n L_{ci} \tag{3.9}$$

where t_i is the thickness in [mm], L_{ci} is the cutting length in [mm].

3.7. Total cost function. The total cost function is defined according to (3.1). Taking k_m = 0.5÷1 \$/kg, k_f =0 ÷1 \$/min, the k_f/k_m ratio varies between 0 - 2 kg/min. If k_f/k_m= 0, we get the mass minimum. k_f/k_m = 2.0 means a very high labor cost (Japan, USA), k_f/k_m = 1.5 and 1.0 mean a West European labor cost, k_f/k_m = 0.5 means the labor cost in developing countries.

4. Welded stiffened plate

4.1. Main data for the optimization. The cost function is calculatedaccording to (3.1), where $A = b_0 t_f + \varphi h_s t_s, \Theta_d = 3$, $\kappa = \varphi + 1$, $L_w = 2L\varphi$ and φ is the number of stiffeners. The stiffeners are welded to the plate by double fillet welds.

The main data for the optimization are as follows:

The Young modulus of the steel is $E = 2.1 * 10^5$ MPa, the density is $\rho = 7.85 * 10^{-6}$ kg/mm^3, the Poisson ratio is $\nu = 0.3$, the yield stress is f_y = 235 MPa, the width of the plate is b_0 = 4200 mm and the plate length is L = 4000 mm.

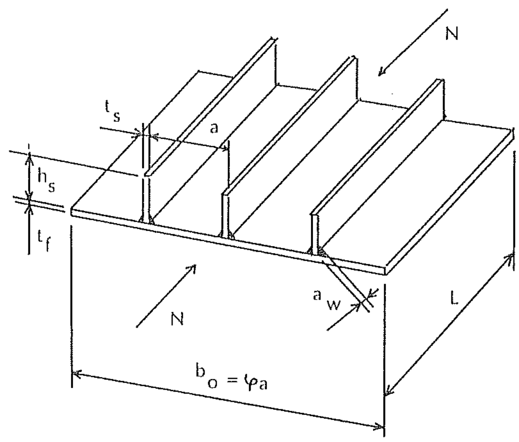

Figure 8. Stiffened plate

The compression force is

$$N = f_y b_0 t_{f\,\max} = 235 * 4200 * 20 = 1.974 * 10^7 \quad \text{(N)} \tag{4.1}$$

The independent design variables are as follows (Figure 8): The plate thickness t_f), the height h_s and thickness t_s of the stiffeners and the number of stiffeners $\varphi = b_0/a$.

4.2. Design constraints.

a) According to API [13, 1987] the overall buckling constraint for the compressed plate with uniform distance stiffeners is (Figure 8)

$$N \leq \chi f_y A \tag{4.2}$$

where the buckling factor χ is a function of the reduced slenderness factor $\bar{\lambda}$:

$$\chi = \begin{cases} 1 & \text{if } \ \bar{\lambda} \leq 0.5 \\ 1.5 - \bar{\lambda} & \text{if } \ 0.5 < \bar{\lambda} \leq 1 \\ 0.5/\bar{\lambda} & \text{if } \ \bar{\lambda} > 1 \end{cases} \tag{4.3}$$

The factor $\bar{\lambda}$ is given by

$$\bar{\lambda} = \frac{b_0}{t_f}\sqrt{\frac{12(1-\nu^2)f_y}{E\pi^2 k_{\min}}} \tag{4.4}$$

in which

$$k_{\min} = \min(k_F, k_R) \tag{4.5}$$

$$k_R = 4\varphi^2\,, \qquad k_F = \begin{cases} \dfrac{(1+\alpha^2)^2 + \varphi\gamma}{\alpha^2(1+\varphi\delta_P)} & \text{if} \quad \alpha = \dfrac{L}{b_0} \leq 4\sqrt{1+\varphi\gamma} \\[2ex] \dfrac{2(1+\sqrt{1+\varphi\gamma})}{1+\varphi\gamma} & \text{if} \quad \alpha =\geq 4\sqrt{1+\varphi\gamma} \end{cases} \tag{4.6}$$

and

$$\delta_P = \frac{h_s t_s}{b_0 t_f}, \qquad \gamma = \frac{E I_s}{b_0 D}, \qquad I_s = \frac{h_s^3 t_s}{3}, \qquad D = \frac{E t_f^3}{12(1-\nu^2)} . \tag{4.7}$$

Equation $(4.7)_2$ can be rewritten as

$$\gamma = 4(1-\nu^2)\frac{h_s^3 t_s}{b_0 t_f^3} = 3.64\frac{h_s^3 t_s}{b_0 t_f^3} , \tag{4.8}$$

where I_s is the moment of inertia of one stiffener about an axis parallel to the plate surface at the base of the stiffener, D is the torsional stiffness of the main plate.

Optimization was made using Hillclimb technique [14, 1989].

Table 2. Optimum rounded sizes of welded stiffened plates in mm with fillet welds using different welding technologies for $k_f/k_m = 2.0$

Welding technology	k_f/k_m	h_s	t_f	φ	t_s	ρV(kg)	K/k_m(kg)
Same for each technology	0.0	210	17	13	11	2737	2737
	0.5	230	17	6	19	3242	6313
SMAW	1.0	235	17	6	19	3258	9409
	1.5	235	17	6	19	3258	12484
	2.0	235	17	6	19	3258	15559
	0.5	230	17	6	19	3242	5749
SMAW HR	1.0	230	17	6	19	3242	8257
	1.5	230	17	6	19	3242	10764
	2.0	235	17	6	19	3258	13306
	0.5	230	17	6	19	3242	5553
FCAW-MC	1.0	230	17	6	19	3242	7864
	1.5	230	17	6	19	3242	10175
	2.0	235	17	6	19	3258	12521
	0.5	230	17	6	19	3242	5299
GMAW-C	1.0	230	17	6	19	3242	7357
GMAW-M	1.5	235	17	6	19	3258	9444
	2.0	230	17	6	19	3242	11471
SAW	0.5	230	17	6	19	3242	5064
ISW	1.0	230	17	6	19	3242	6886
FCAW	1.5	230	17	6	19	3242	8707
	2.0	235	17	6	19	3258	10564

b) The buckling constraint of the stiffener is

$$\frac{h_s}{t_s} \le \frac{1}{\beta_s} = 14\sqrt{\frac{235}{f_y}} . \tag{4.9}$$

The size ranges for the variables are as follows:

$t_f = 6 \div 20$ mm, $\quad h_s = 84 \div 280$ mm, $\quad t_s = 6 \div 25$ mm and $\quad \varphi = 4 \div 15$ mm.

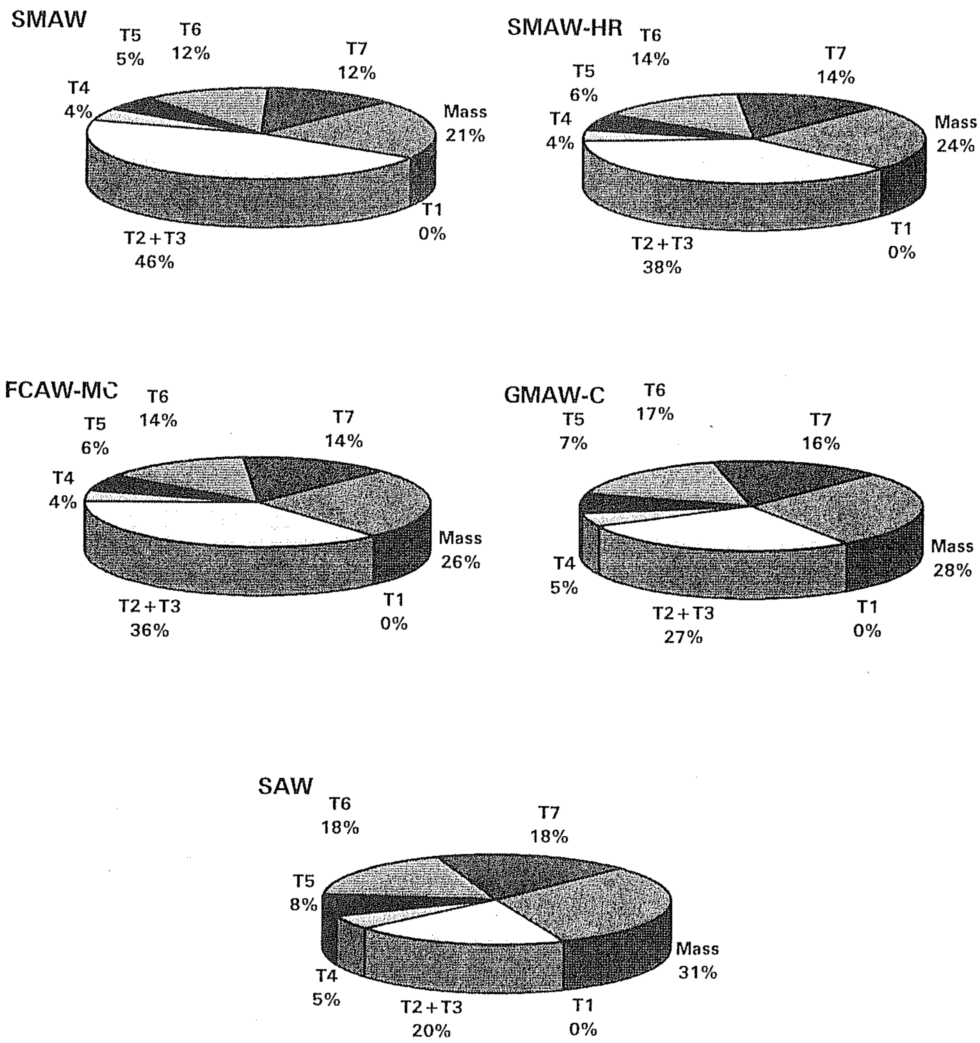

Figure 9. The total cost distribution of the welded stiffened plate with fillet welds using different welding technologies for k_f/k_m=2.0

The elements of cost function for the welded stiffened plate are as follows:

Size of welded joint	$a_w = t_s$	Cross section area	$A = b_0 t_f + \varphi h_s t_s$
Material cost	$\rho V = \rho L A$	Fabrication costs	$k_f/k_m \sum_i T_i$

Further data

Formula for T_i	Data for T_i
$T_1 = C_1\delta\sqrt{\kappa\rho V}$	$\rho = 7.85 * 10^{-6}$, $C_1 = 1$, $\kappa = \varphi + 1$, $\Theta_d = 2$
$T_2 + T_3 = 1.3\sum C_{2i}a_{wi}^2 L_{wi}$	$C_{2i} = 0.7889 * 10^{-3}$ for SMAW and $L_{wi} = 2L\varphi$, L in mm
$T_4 = \Theta_{de}\left(a_e + b_e t^3 + \frac{1}{a_c t^4}\right)A_p$	$a_e = 9.2 * 10^{-4}$, $b_e = 4.15 * 10^{-7}$, $t = t_s$, or t_f , $Ap = \varphi h_s L$ or $b_0 L$
$T_5 = \Theta_{ds}a_{sp}A_s = 5 * 10^{-7}$	$a_{sp} = 3 * 10^{-6}$, $A_s = \varphi h_s L + b_0 L$
$T_6 = \Theta_{dp}(a_{gc} + a_{tc})A_s$	$a_{gc} = 3 * 10^{-6}$, $a_{tc} = 4.15 * 10^{-6}$ and $A_s = \varphi h_s L + b_0 L$
$T_7 = \sum C_{7i}t_i^n L_{ci}$	$C_7 = 1.1388$, $t = t_s$ or t_f , $n = 0.25$, $L_{ci} = (h_s + L)$ or $(b_0 + L)$

Table 2 shows the optimum discrete sizes of the stiffened plate made with different welding technologies. Figure 9 shows the distribution of the total cost. The diagrams illustrate that this distribution depends on the welding technologies, the type of welding, the ratio of material and fabrication specific costs and the structure type as well.

The welding technologies in Figure 9 are given in decreasing order related to the welding time and cost. The differences between them are great. The welding time and cost are the greatest for SMAW, the quickest and cheapest are SAW, FCAW and ISW. For stiffened plates using SMAW, 46% of the total cost is the welding cost, using SAW, it is only 20%. The fabrication costs of stiffened plates have a larger ratio in total cost, because stiffened plates contain more elements, which need more welding time.

The mass of stiffened plate is ρLA = 3258 kg (Table 2), the fabrication cost is 100 (15559-3258) / 15559 = 79 % of the total cost. Cost savings can be achieved using a cheaper welding technology, like SAW instead of SMAW or GMAW, if it is possible. Table 3 shows the cost savings for the two different structures and for the five different groups of welding. For stiffened plates the cost savings can be 32 % of the total cost. All compared results are optimized.

Table 3. Cost savings for different welding technologies

Welding technology k_f/k_m=2.0	Total cost	Cost savings in %
SMAW	15559	0
SMAW-HR	13305	14
FCAW-MC	12521	20
GMAW-C	11471	27
SAW	10560	32

5. Ship deck optimization

5.1. Sructural elements. Cellular plates consist of two face sheets and a grid of ribs welded between them. The main advantage of such a plate structure is that the cells have a large torsional stiffness, which allows designers to construct plates of small height. The disadvantage of cellular plates lies in fabrication difficulty, since, when the height is smaller than 800÷1000 mm, it is impossible to weld the ribs to the face sheets from inside.

Some applications of cellular plates are as follows: double bottoms of ships, rudders of ships, floating roofs of cylindrical storage tanks, box gates for dry docks, wings of aircraft structures, bridge decks, floating bridges, offshore platforms, elements of machine tool structures (press tables, mounting desks, base plates), mining shields, floors of buildings, lightweight roofs, etc.

Regarding the fabrication of cellular plates there are several possibilities to join the ribs to the face sheets. The simplest but not the cheapest solution is to use faceplate elements and weld them to ribs from outside by fillet welds. Special welds such as arc-spot welds, slot or plug welds as well as electron-beam or laser welds can be used without cutting larger face sheet parts. A combination of fillet and arc-spot welds is shown in Figure 10.

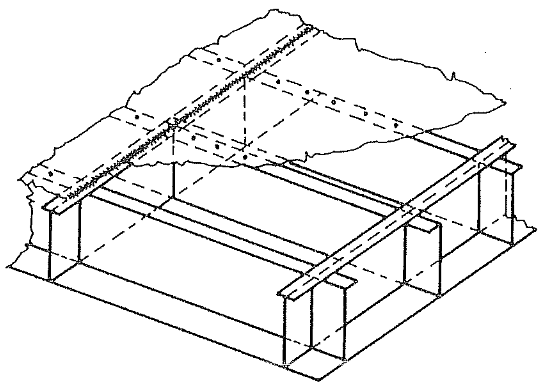

Figure 10. Cellular plate with a combination of fillet and arc spot welds

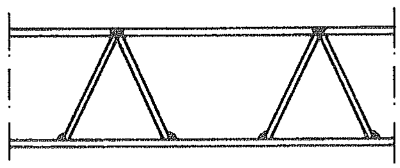

Figure 11. A special cellular plate with longitudinal stiffeners proposed by Suruga and Maeda [15, 1976]

Suruga and Maeda proposed a special cellular plate construction for bridge decks (Figure 11), but this solution is too expensive.

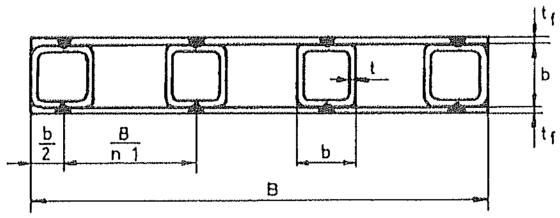

Figure 12. Cross-section of the ship deck panel

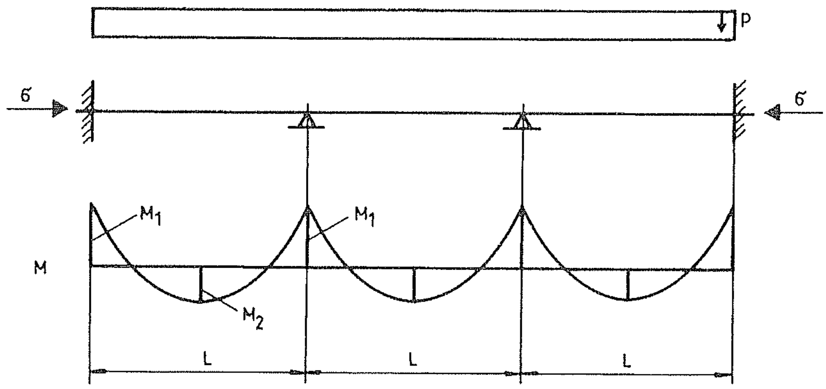

Figure 13. Bending moment diagram of the ship deck panel

An interesting application of cellular plates is the ship deck panels. The main specialties of this application are as follows: 1) only longitudinal ribs of square hollow section (SHS) are used joined to the face sheets by arc-spot welding, thus, in the cost function the fabrication cost of arc-spot welds should be included; 2) to avoid the vibration resonance, the first eigenfrequency of the plate should be larger than a prescribed value.

The aim of the present study is to work out a minimum cost design of such cellular plates considering, in addition to the stress constraint, the eigenfrequency constraint as well, and the fabrication cost of arc-spot welds.

5.2. The cost function. The cross-section of the deck panel is shown in Figure 12. The cellular plate consists of two face sheets of thickness t_f and longitudinal SHS ribs of number n with dimensions of b and t.

In the longitudinal direction the plate ends are clamped and the panel is supported in two points, thus, it can be calculated as a three-span beam (Figure 13) loaded axially with a compression stress $\sigma = N/A_{eff}$, A_{eff} being the effective cross-section for compression (Figure 15), and transversely by a uniformly distributed normal load of a factored intensity p.

The cost is calculated according to (3.1). The volume of the structure is

$$V = 3l(nA_{SHS} + 2Bt_f)\,. \tag{5.1}$$

Considering the corner roundings according to a formula given by DASt [16, 1986], the cross-sectional area of a SHS is approximately

$$A_{SHS} = 0.99 * 4(b-t)t\left(1 - 0.43\frac{t}{b-3t}\right)\,. \tag{5.2}$$

The fabrication times are as follows. The time of preparation, assembly and tacking can be expressed as

$$T_1 = C_1 \Theta_d (\kappa \rho V)^{0.5}\,, \tag{5.3}$$

where $C_1 = 1.0\ \ \mathrm{min/kg}^{0.5}$ is the difficulty factor expressing the effect of the type of structure (planar or spatial), κ is the number of assembled structural elements, in our case $\kappa = n + 2$.

The time of arc-spot welding is given by

$$T_2 = n_s T_s \tag{5.4}$$

where n_s is the number of spots, T_s is the time of welding of one spot weld and of the electrode transfer to the next spot.

The additional time for deslagging, chipping and changing the electrode can be calculated as

$$T_3 = 0.3T_2\,. \tag{5.5}$$

Since data for T_s cannot be found in literature, we take $T_s = 0.3$ min noting that it depends on the welding equipment and the degree of automation.

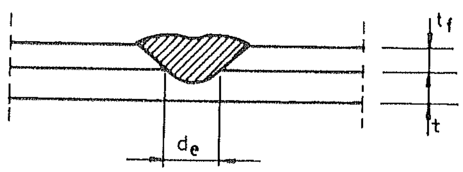

Figure 14. Effective diameter of an arc-spot weld

The number of spots can be calculated by means of the spot pitch a. The required minimum spot pitch can be determined considering a spot weld as a pin [17, 1978], [18, 1990].

Limiting forces for a pin, according to Eurocode 3 (EC3) [19, 1992] are as follows:

– for bearing

$$F_b = 1.5 t_f d_e f_y / \gamma_{Mp} \tag{5.6}$$

with $d_e = 2t_f$ (Figure 14) and $\gamma_{Mp} = 1.25$; $F_b = 2.4 t_f^2 f_y$

– for shear

$$F_Q 0.6 t \frac{\pi d_e^2}{4} \frac{f_u}{\gamma_{Mp}} = 1.508 t_f^2 t f_u \,. \tag{5.7}$$

For steel Fe 360 the ultimate strength is $f_u = 360$ and the yield stress is $f_y = 235$ MPa, for steel Fe 510 they are $f_u = 510$ and $f_y = 355$ MPa.

The spot weld is loaded by the force FW from the shear acting in a bent beam

$$F_w = \frac{Q S_\xi}{I_\xi} a \tag{5.8}$$

where Q is the shear force, S_ξ and I_ξ are the first moment and moment of inertia of an effective cross-section as shown in Figure 16 and given by (5.21), respectively, while a is the spot pitch. From the condition one obtains the required maximum spot pitch

$$a_{\max} = \frac{F_{b,Q} I_\xi}{Q S_\xi} \qquad \text{but} \qquad a_{\max} \le 50 t_f \,. \tag{5.9}$$

The number of spots in (5.4) can be expressed as

$$n_s = 6nL/a \,. \tag{5.10}$$

5.3. Constraint on eigenfrequency. A serviceability constraint can be defined expressing that the first eigenfrequency of a simply supported bent beam of span length L should be larger than a prescribed value

$$f_1[\text{Hz}] = \frac{\pi}{2L^2} \left(\frac{10^3 E I_x}{m} \right)^{1/2} \ge f_0 \,, \tag{5.11}$$

where E is the modulus of elasticity and I_x is the moment of inertia of the whole cross-section:

$$I_x = n I_{SHS} + B t_f (b + t_f)^2 / 2 \,. \tag{5.12}$$

According to DASt (1986) the moment of inertia of a SHS is approximately

$$I_{SHS} = \frac{2}{3} (b - t)^3 t \left(1 - 0.86 \frac{t}{b - 3t} \right) \,. \tag{5.13}$$

In the formula for the mass m an additive mass m_{add} should be involved, thus

$$m = \rho (n A_{SHS} + 2 B t_f) + m_{add} \,. \tag{5.14}$$

It should be mentioned that f_1 is larger than the value obtained from the formula (5.11) because the beam is clamped and not simply supported. In spite of that, one can use the above approximation since it is obvious from Table 1 that this constraint is not active.

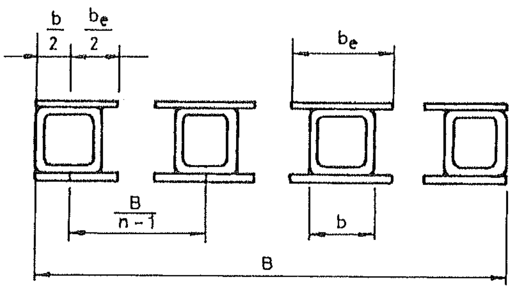

Figure 15. Effective cross-section for compression

5.4. Constraint on stress due to compression and bending. According to EC3, the stress constraint should be defined for a section of class 4 as follows:

$$\frac{N}{\chi A_{eff} f_{y1}} + \frac{k_x \psi M_1}{W_\xi f_{y1}} \leq 1 \tag{5.15}$$

where χ is the overall buckling factor and

$$\chi = \frac{1}{\Phi + (\Phi^2 - \bar{\lambda}^2)^{1/2}}, \qquad \Phi = 0.5\left[1 + 0.34(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right], \qquad \bar{\lambda} = \frac{KL}{\lambda_1 r}\beta_A^{1/2}. \tag{5.16}$$

Here K depends on the supports – for a beam with clamped ends $K = 0.5$ – and

$$\lambda_1 = \pi(E/f_y)^{1/2}\beta_A^{1/2}, \qquad r = (I_{eff}/A_{eff})^{1/2}, \qquad \beta_A = \frac{A_{eff}}{nA_{SHS} + 2Bt_f}. \tag{5.17}$$

To obtain the effective cross-section, the effective width of face sheets should be calculated according to EC3

$$b_e = \rho_P \frac{B}{n-1}, \qquad \bar{\lambda}_P = \frac{B/\left[(n-1)t_f\right]}{28.4\varepsilon k_\sigma^{1/2}}, \qquad \varepsilon = \sqrt{\frac{235}{f_y}} \tag{5.18}$$

with

$$k_\sigma = 4, \qquad \bar{\lambda}_P = \frac{B}{56.8\varepsilon(n-1)t_f} \tag{5.19}$$

where

$$\rho_P = \begin{cases} 1 & \text{if } \bar{\lambda}_P \leq 0.673 \\ \dfrac{1}{\bar{\lambda}_P} - \dfrac{0.22}{\bar{\lambda}^2} & \text{if } \bar{\lambda}_P \geq 0.673 \end{cases}. \tag{5.20}$$

Considering the effective cross-section shown in Figure 15 we get

$$A_{eff} = nA_{SHS} + 2B_e t_f, \qquad B_e = b + (n-1)b_e, \qquad I_{eff} = nI_{SHS} + B_e t_f (b + t_f)^2/2. \tag{5.21}$$

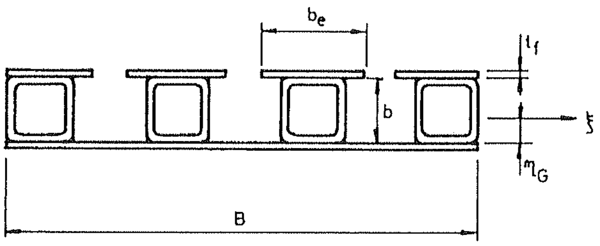

Figure 16. Effective cross-section for bending

According to the moment diagram shown in Figure 13

$$M_1 = BpL^2/12 \,. \tag{5.22}$$

This bending moment should by multiplied by a dynamic factor

$$k_x = 1 - \frac{\mu_x N}{\chi(nA_{SHS} + 2Bt_f)f_y}, \qquad \text{but} \qquad k_x \le 1.5 \tag{5.23}$$

$$\mu_x = \bar\lambda(2\beta_M - 4), \qquad \text{but} \qquad \mu_x \le 0.9 \,. \tag{5.24}$$

For our case $\beta_M = 1.3$ and $\mu_x = -1.4\,\bar\lambda$, thus

$$k_x = 1 + \frac{1.4\bar\lambda\beta_A N}{\chi A_{eff} f_y} \,. \tag{5.25}$$

For bending another asymmetric effective cross-section should be taken into account as shown in Figure 16. The distance of gravity centre G is

$$\eta_G = \frac{nA_{SHS}(b+t_f)/2 + B_e t_f(b+t_f)}{nA_{SHS} + (B+B_e)t_f} \,. \tag{5.26}$$

The moment of inertia is given by

$$I_\xi = nI_{SHS} + nA_{SHS}\left(\frac{b+t_f}{2} - \eta_G\right)^2 + Bt_f\eta_G + Bt_f(b+t_f-\eta_G)^2 \,. \tag{5.27}$$

The first moment of the cross sectional area for the calculation of (5.18) and (5.26) and the corresponding section modulus are

$$S_\xi = b_e(b+t_f-\eta_G) \qquad \text{and} \qquad W_\xi = \frac{I_\xi}{b+t_f-\eta_G}, \tag{5.28}$$

respectively.

5.5. The optimization procedure. In the minimum cost design the optimum values of b, t, t_f and n are sought, which minimize the cost function (3.1) and fulfil the design constraints (5.11) and (5.15). In the first phase the above mentioned variables are treated as continuous ones and the optima are determined using the Rosenbrock's hillclimb mathematical programming method. In the second phase the discrete values of variables are calculated using a complementary search method. In this search the minimum values are taken as

$b_{min} = 30, t_{min} = 2, t_{fmin} = 2$ mm and $n_{min} = 4$.

The discrete values of SHS are sought according to the pre-standard prEN 10219-2 [20, 1992].

The numerical data are as follows: $f_0 = 18$ Hz, $E = 2.1 * 10^5$ MPa, $B = 2000$, $L = 2250$ mm, $\rho = 7850$ kg/m^3 = 7.85*10^{-6} kg/mm^3, $m_{add} = 2 * 50 = 100$ kg/m = 0.1 kg/mm, $p = 3.5$ kN/m^2 = $3.5 * 10^{-3}$ N/mm^2, $\psi = 1.4$, $\sigma = N/A_{eff} = 150$ MPa.

The computational results are summarized in Table 4.

Table 4. Optimization results: optimum dimensions in mm, number of ribs n, fulfilling the design constraints (5.11) and (5.15), as well as K/k_m - values in kg for cost in function of the ratio k_f/k_m

f_y [MPa]	b	t	t_f	n	(5.11)	(5.15)	K/k_m		
							$\frac{k_f}{k_m} = 0$	$\frac{k_f}{k_m} = 1$	$\frac{k_f}{k_m} = 2$
235	60	2	2	4	31.5>18Hz	0.99<1	520	898	1276
355	40	2	2	4	21.1>18Hz	0.89<1	486	859	1231

As can be seen from Table 1, the number of ribs and the thickness of face sheets should be minimum to achieve minimum cost. For larger yield stress the dimension of SHS can be decreased, thus the cost is also smaller. It can also be seen that the eigenfrequency constraint (5.11) is passive and the stress constraint (5.15) is active.

The optimum dimensions do not depend on the fabrication cost or on the ratio of k_f/k_m . In fabrication cost only the distance of spots depends on the structural dimensions (see 5.9), but, in all cases, the limit $a_{max} = 50t_f$ is governing, constraint (5.9) gives much larger values for a. Since $t_f = t_{fmin} = 2$ mm for all cases the fabrication cost remains the same. The fabrication cost is quite high for $f_y = 235$ MPa, in the case of $k_f/k_m = 1$ it is 100(898-520)/898 = 42% and for $k_f/k_m = 2$ it is 100(1276-520)/1276 = 59% of the whole cost.

6. Conclusions

We have shown that stiffened plates play an important role in structural design. The analysis of these structures can be made for static loading and can be built into the optimization software. Cost calculation of these structures is important due to the high volume of welding. Two examples show how we can perform cost minimization using different material and fabrication cost factors. The optima of stiffened plates with compression load show that for material cost the number of stiffeners is high (13 stiffeners), for high fabrication cost the number is low (6 stiffeners). For different welding technologies, optima are different. For SMAW the cost of welding is close to half of the total cost, for SAW the welding cost is only 20% of the total cost. For the second example where the stiffeners are square hollow sections, not only the stress and stability constraints (taking into account the effective width due to bending and compression), but the eigenfrequency constraints are also considered. In most cases the optima are determined by the thickness lower limits and the stability constraint. The fabrication cost can be more than half the total cost at the optimum.

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